Disclaimer: This pdf can be used or distributed for non-profit purpose. It is against owners will that you use this for profit making or any other source(s) of income.

2014 Jan C34

4.

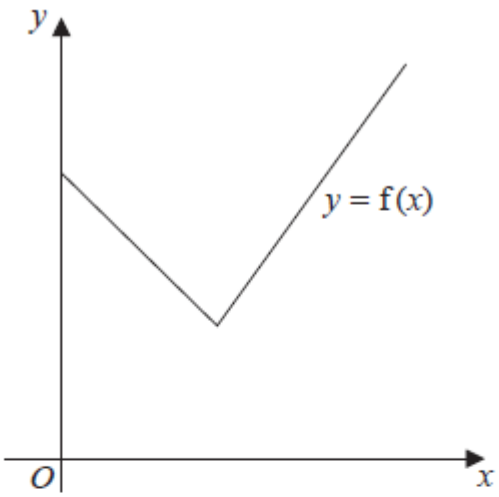

Figure 1

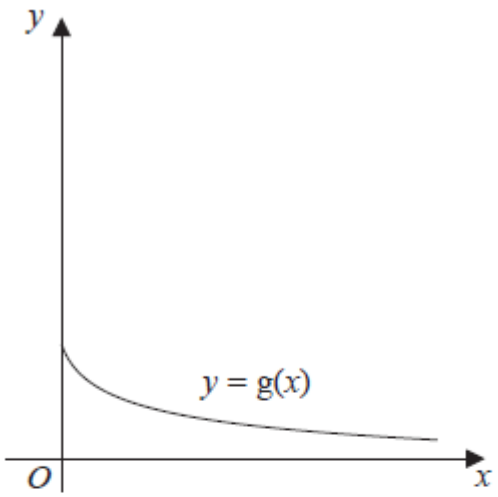

Figure 2

Figure 1 shows a sketch of part of the graph $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2|3 - x| + 5, \quad x \geqslant 0$$

Figure 2 shows a sketch of part of the graph $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x + 9}{2x + 3}, \quad x \geqslant 0$$

(a) Find the value of fg(1)

(2)

(b) State the range of g

(2)

(c) Find $\mathrm{g}^{-1}(x)$ and state its domain.

(4)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(d) state the range of possible values of k.

(3)

2014 June C34

7. The function f is defined by

$$f: x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, x \neq -1$$

(a) Find an expression for $f^{-1}(x)$

(3)

(b) Show that

$$ff(x) = \frac{x+a}{x-1}, \quad x \in \mathbb{R}, x \neq -1, x \neq 1$$

where a is an integer to be determined.

(4)

The function g is defined by

$$g: x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, 0 \leqslant x \leqslant 5$$

(c) Find the value of fg(2)

(2)

(d) Find the range of g

(3)

2015 Jan C34

3. The function g is defined by

$$g : x \mapsto |8 - 2x|, \qquad x \in \mathbb{R}, \qquad x \geqslant 0$$

(a) Sketch the graph with equation $y = g(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(3)

(b) Solve the equation

$$|8 - 2x| = x + 5$$

(3)

The function f is defined by

$$f : x \mapsto x^2 - 3x + 1, \qquad x \in \mathbb{R}, \qquad 0 \leqslant x \leqslant 4$$

(c) Find fg(5).

(2)

(d) Find the range of f. You must make your method clear.

(4)

2015 June C34

3.

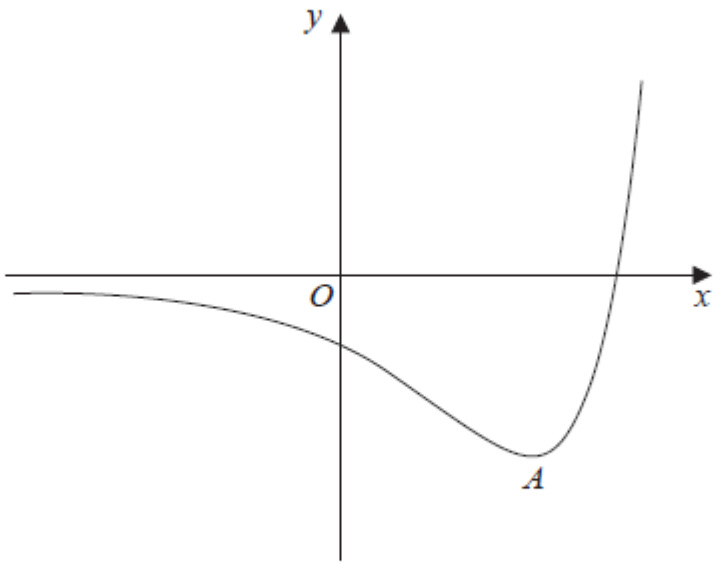

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (2x - 5)e^x, \qquad x \in \mathbb{R}$$

The curve has a minimum turning point at A.

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $f(x) = k$, where k is a constant, has exactly two roots,

(b) state the range of possible values of k.

(2)

(c) Sketch the curve with equation $y = |f(x)|$.

Indicate clearly on your sketch the coordinates of the points at which the curve crosses or meets the axes.

(3)

2016 Jan C34

11.

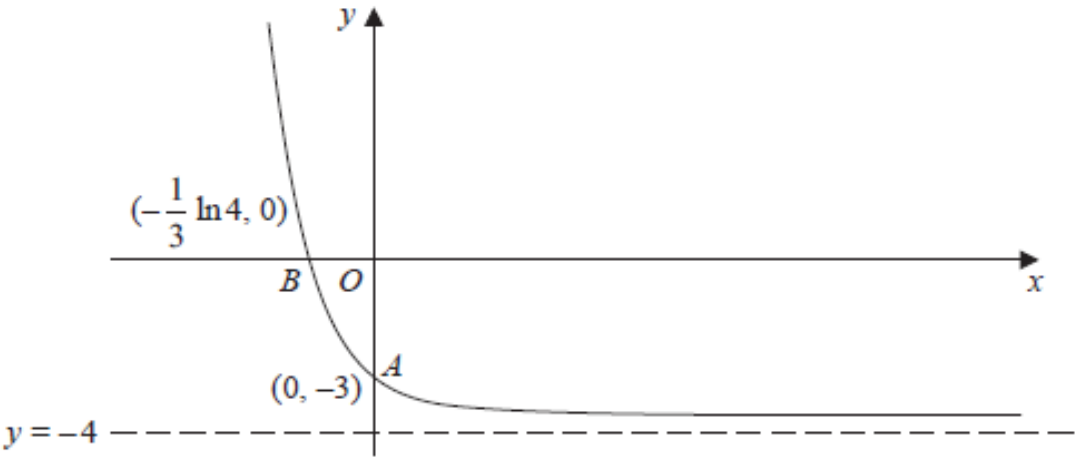

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x), \quad x \in \mathbb{R}$

The curve meets the coordinate axes at the points $A(0, -3)$ and $B(-\frac{1}{3}\ln 4, 0)$ and the curve has an asymptote with equation $y = -4$

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$

(4)

(b) $y = 2f(x) + 6$

(3)

On each sketch, give the exact coordinates of the points where the curve crosses or meets the coordinate axes and the equation of any asymptote.

Given that

$$f(x) = e^{-3x} - 4, \qquad x \in \mathbb{R}$$

$$g(x) = \ln\left(\frac{1}{x+2}\right), \qquad x > -2$$

(c) state the range of f,

(1)

(d) find $f^{-1}(x)$,

(3)

(e) express $fg(x)$ as a polynomial in x.

(3)

2016 June C34

6. Given that a and b are constants and that $a > b > 0$

(a) on separate diagrams, sketch the graph with equation

(i) $y = |x - a|$

(ii) $y = |x - a| - b$

Show on each sketch the coordinates of each point at which the graph crosses or meets the x-axis and the y-axis.

(5)

(b) Hence or otherwise find the complete set of values of x for which

$$|x - a| - b < \frac{1}{2}x$$

giving your answer in terms of a and b.

(4)

2017 Jan C34

4. Given that

$$f(x) = \frac{4}{3x+5}, \quad x > 0$$

$$g(x) = \frac{1}{x}, \quad x > 0$$

(a) state the range of f,

(2)

(b) find $f^{-1}(x)$,

(3)

(c) find $fg(x)$.

(1)

(d) Show that the equation $fg(x) = gf(x)$ has no real solutions.

(4)

2017 Jan C34

7.

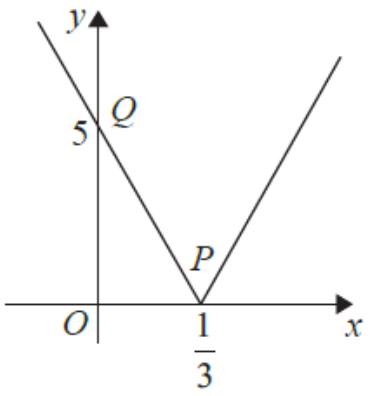

Figure 2

Figure 2 shows a sketch of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The point $P\left(\frac{1}{3}, 0\right)$ is the vertex of the graph.

The point $Q(0, 5)$ is the intercept with the y-axis.

Given that $\mathrm{f}(x) = |ax + b|$, where a and b are constants,

(a) (i) find all possible values for a and b,

(ii) hence find an equation for the graph.

(4)

(b) Sketch the graph with equation

$$y = \mathrm{f}\left(\frac{1}{2}x\right) + 3$$

showing the coordinates of its vertex and its intercept with the y-axis.

(3)

2017 June C34

3. The function g is defined by

$$g(x) = \frac{6x}{2x+3} \qquad x > 0$$

(a) Find the range of g.

(2)

(b) Find $g^{-1}(x)$ and state its domain.

(3)

(c) Find the function $gg(x)$, writing your answer as a single fraction in its simplest form.

(3)

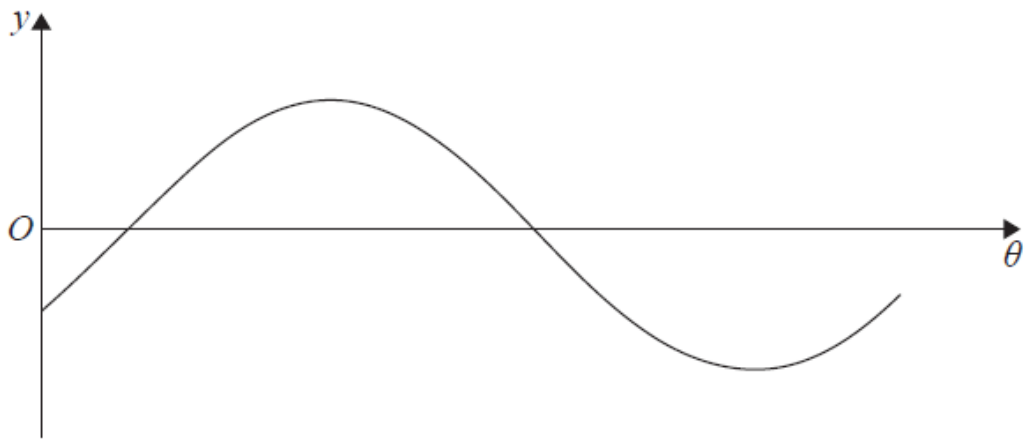

Figure 3

Figure 3 shows a sketch of the graph with equation $y = 2\sin\theta - \cos\theta, \quad 0 \leqslant \theta < 360°$

(b) Sketch the graph with equation

$$y = |2\sin\theta - \cos\theta|, \qquad 0 \leqslant \theta < 360°$$

stating the coordinates of all points at which the graph meets or cuts the coordinate axes.

(3)

2017 Oct C34

9.

$$f(x) = 2\ln(x) - 4, \quad x > 0, \quad x \in \mathbb{R}$$

(a) Sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(5)

(b) Find the exact solutions of the equation $|f(x)| = 4$

(4)

$$g(x) = e^{x+5} - 2, \quad x \in \mathbb{R}$$

(c) Find $gf(x)$, giving your answer in its simplest form.

(3)

(d) Hence, or otherwise, state the range of gf.

(1)

2018 Jan C34

4.

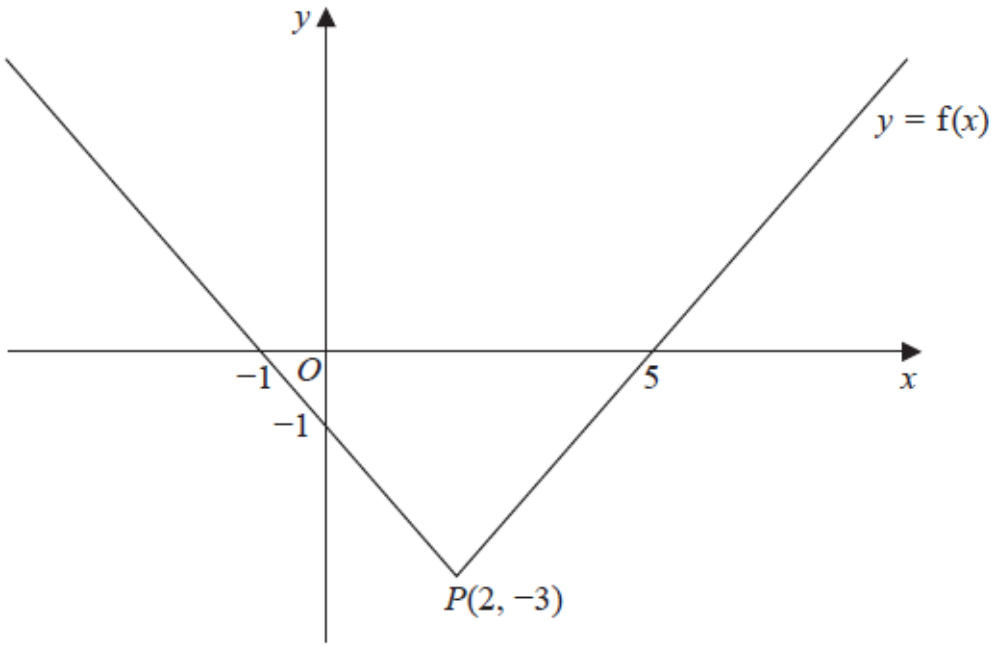

Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = \mathrm{f}(x), \quad x \in \mathbb{R}$

The graph consists of two half lines that meet at the point $P\,(2, -3)$, the vertex of the graph.

The graph cuts the y-axis at the point $(0, -1)$ and the x-axis at the points $(-1, 0)$ and $(5, 0)$.

Sketch, on separate diagrams, the graph of

(a) $y = \mathrm{f}(|x|)$,

(3)

(b) $y = 2\mathrm{f}(x + 5)$.

(3)

In each case, give the coordinates of the points where the graph crosses or meets the coordinate axes.

Also give the coordinates of any vertices corresponding to the point P.

2018 Jan C34

10. It is given that

$$f(x) = e^{-2x} \qquad x \in \mathbb{R}$$

$$g(x) = \frac{x}{x-3} \qquad x > 3$$

(a) Sketch the graph of $y = f(x)$, showing the coordinates of any points where the graph crosses the axes.

(2)

(b) Find the range of g

(2)

(c) Find $g^{-1}(x)$, stating the domain of g^{-1}

(4)

(d) Using algebra, find the exact value of x for which $fg(x) = 3$

(4)

2018 June C34

5. (i) The functions f and g are defined by

$$\mathrm{f} : x \to \mathrm{e}^{2x} - 5, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \to \ln(3x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

(3)

(b) Find fg(3), giving your answer in its simplest form.

(2)

(ii) (a) Sketch the graph with equation

$$y = |4x - a|$$

where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

Given that

$$|4x - a| = 9a$$

where a is a positive constant,

(b) find the possible values of

$$|x - 6a| + 3|x|$$

giving your answers, in terms of a, in their simplest form.

(5)

2018 Oct C34

10.

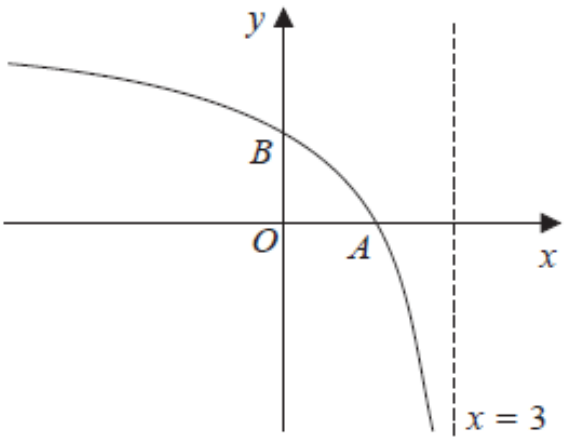

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = g(x)$, where

$$g(x) = \frac{3x - 4}{x - 3}, \quad x \in \mathbb{R}, \quad x < 3$$

The graph cuts the x-axis at the point A and the y-axis at the point B, as shown in Figure 2.

(a) State the range of g.

(1)

(b) State the coordinates of

(i) point A

(ii) point B

(2)

(c) Find $gg(x)$ in its simplest form.

(3)

(d) Sketch the graph with equation $y = |g(x)|$

On your sketch, show the coordinates of each point at which the graph meets or cuts the axes and state the equation of each asymptote.

(3)

(e) Find the exact solution of the equation $|g(x)| = 8$

(3)

2019 Jan C34

3. The function f is defined by

$$\mathrm{f}: x \mapsto 2x^2 + 3kx + k^2 \qquad x \in \mathbb{R},\ -4k \leqslant x \leqslant 0$$

where k is a positive constant.

(a) Find, in terms of k, the range of f.

(4)

The function g is defined by

$$\mathrm{g}: x \mapsto 2k - 3x \qquad x \in \mathbb{R}$$

Given that $\mathrm{gf}(-2) = -12$

(b) find the possible values of k.

(4)

2019 Jan C34

12. Given that k is a positive constant,

(a) sketch the graph with equation

$$y = 2|x| - k$$

Show on your sketch the coordinates of each point at which the graph crosses the x-axis and the y-axis.

(2)

(b) Find, in terms of k, the values of x for which

$$2|x| - k = \frac{1}{2}x + \frac{1}{4}k$$

(3)

2019 June C34

6.

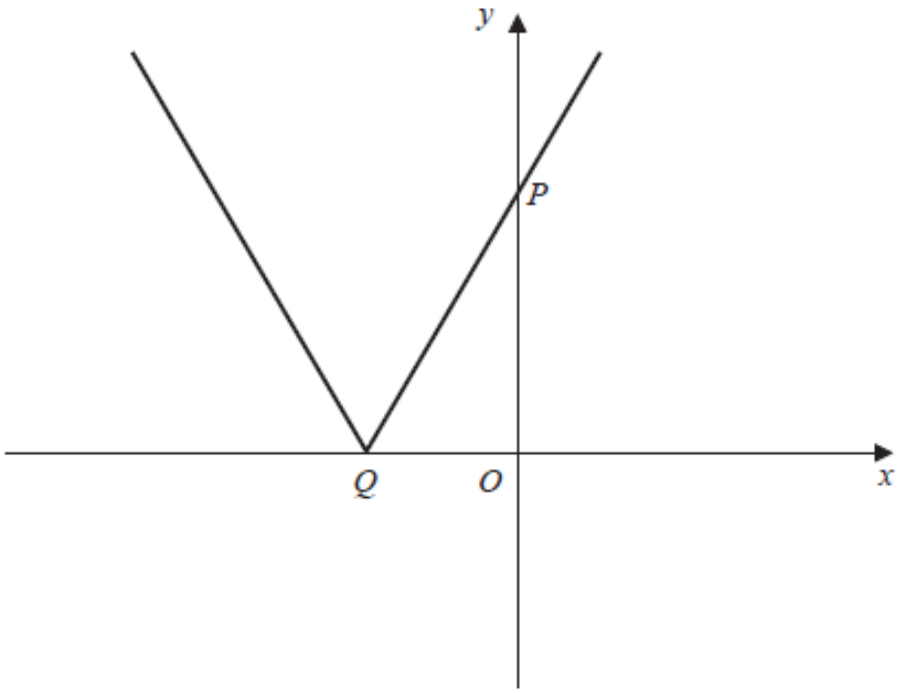

Figure 1

Figure 1 shows a sketch of the graph with equation $y = |4x + 10a|$, where a is a positive constant.

The graph cuts the y-axis at the point P and meets the x-axis at the point Q as shown.

(a) (i) State the coordinates of P.

(ii) State the coordinates of Q.

(2)

(b) A copy of Figure 1 is shown on page 15. On this copy, sketch the graph with equation

$$y = |x| - a$$

Show on the sketch the coordinates of each point where your graph cuts or meets the coordinate axes.

(2)

(c) Hence, or otherwise, solve the equation

$$|4x + 10a| = |x| - a$$

giving your answers in terms of a.

(3)

2019 June C34

8.

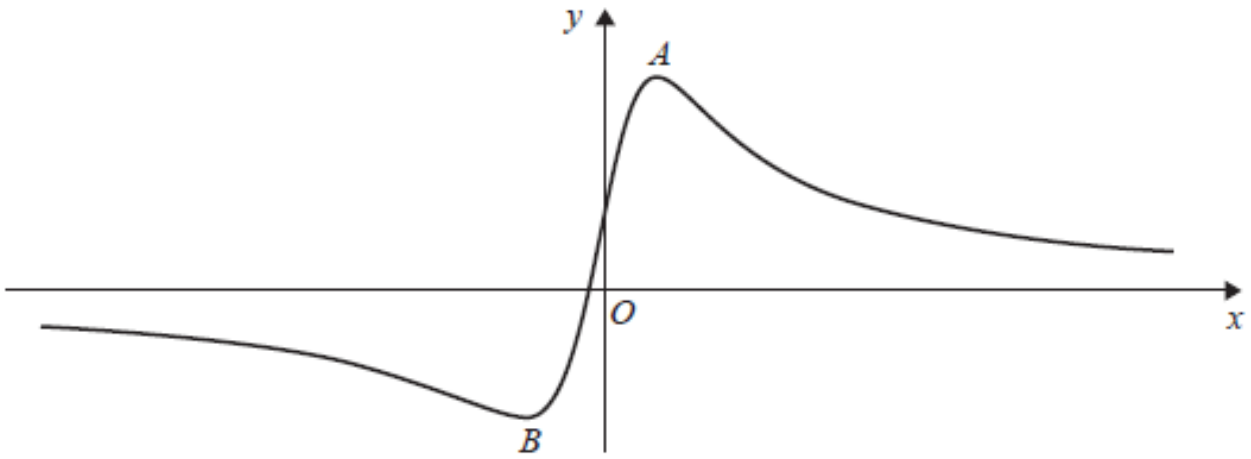

Figure 2

Figure 2 shows a sketch of part of the curve $y = f(x)$, where

$$f(x) = \frac{6x + 2}{3x^2 + 5}, \quad x \in \mathbb{R}$$

(a) Find $f'(x)$, writing your answer as a single fraction in its simplest form.

(3)

The curve has two turning points, a maximum at point A and a minimum at point B, as shown in Figure 2.

(b) Using part (a), find the coordinates of point A and the coordinates of point B.

(4)

(c) State the coordinates of the maximum turning point of the function with equation

$$y = f(2x) + 4 \quad x \in \mathbb{R}$$

(2)

(d) Find the range of the function

$$g(x) = \frac{6x + 2}{3x^2 + 5}, \quad x \leqslant 0$$

(2)

2020 Jan P3

2. The function f and the function g are defined by

$$f(x) = \frac{12}{x+1} \qquad x > 0, x \in \mathbb{R}$$

$$g(x) = \frac{5}{2}\ln x \qquad x > 0, x \in \mathbb{R}$$

(a) Find, in simplest form, the value of $fg(e^2)$

(2)

(b) Find f^{-1}

(3)

(c) Hence, or otherwise, find all real solutions of the equation

$$f^{-1}(x) = f(x)$$

(3)

2020 Jan P3

6.

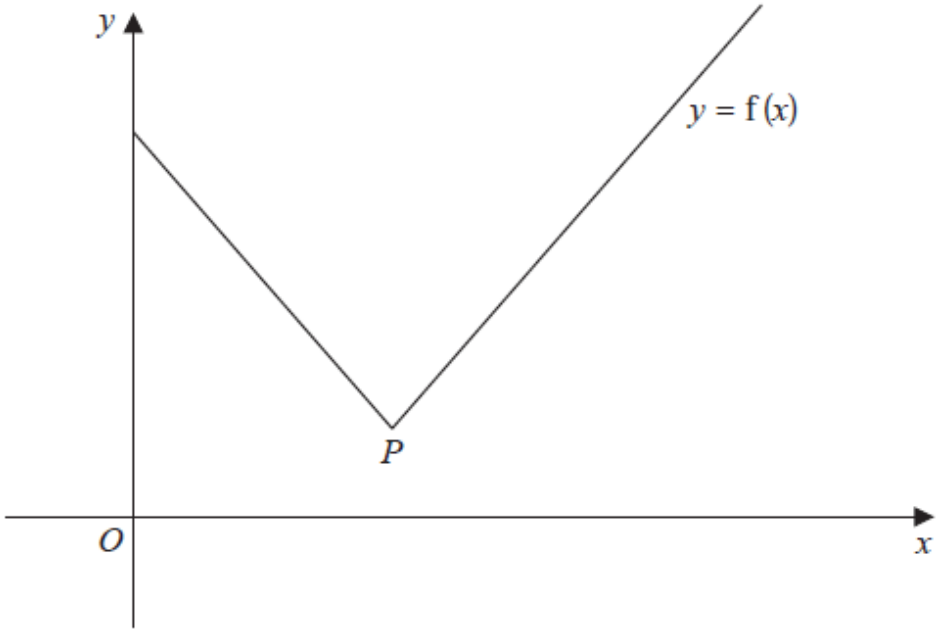

Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = 2|2x - 5| + 3 \qquad x \geqslant 0$$

The vertex of the graph is at point P as shown.

(a) State the coordinates of P.

(2)

(b) Solve the equation $f(x) = 3x - 2$

(4)

Given that the equation

$$f(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k.

(3)

2020 Oct P3

4.

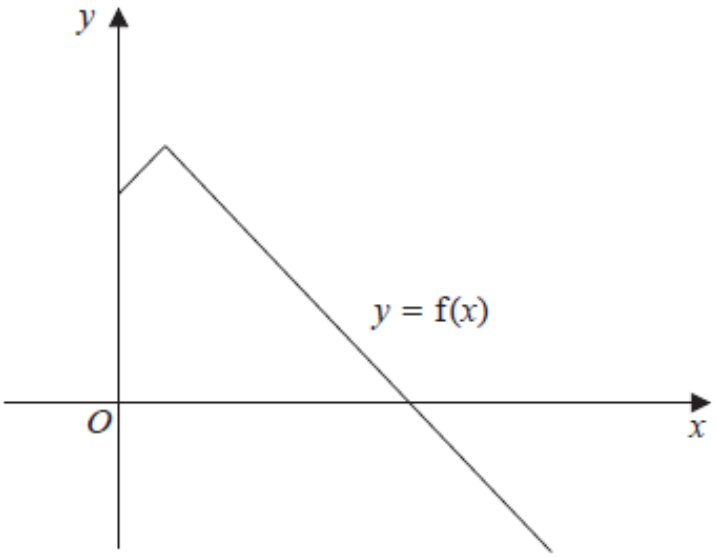

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 21 - 2|2 - x| \qquad x \geqslant 0$$

(a) Find $\mathrm{ff}(6)$

(2)

(b) Solve the equation $\mathrm{f}(x) = 5x$

(2)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has exactly two roots,

(c) state the set of possible values of k.

(2)

The graph with equation $y = \mathrm{f}(x)$ is transformed onto the graph with equation $y = a\mathrm{f}(x - b)$

The vertex of the graph with equation $y = a\mathrm{f}(x - b)$ is $(6, 3)$.

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

2021 Jan P3

2.

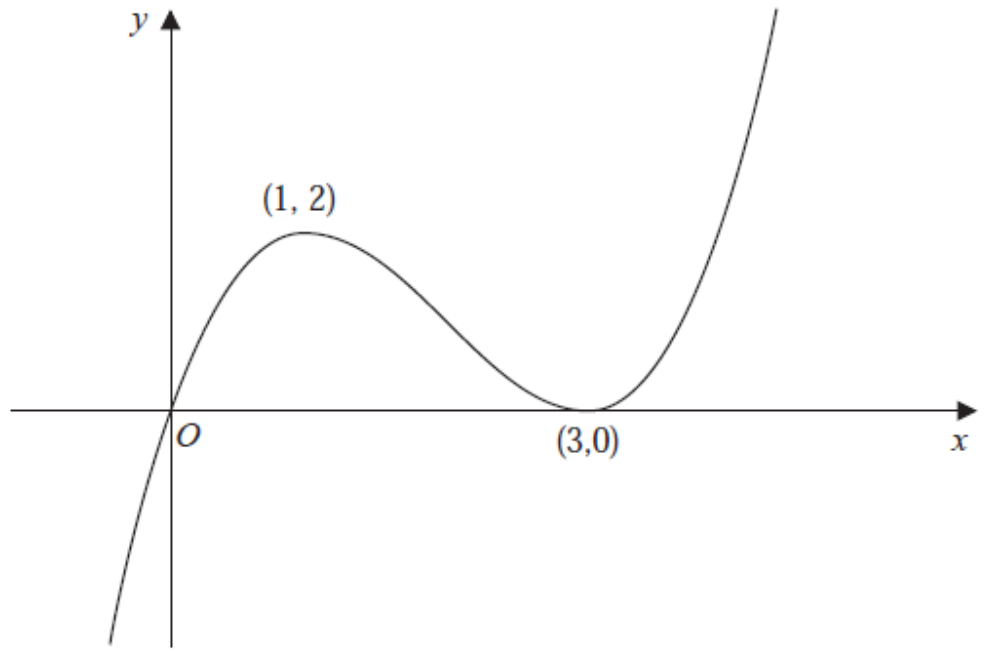

Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, where $x \in \mathbb{R}$ and $f(x)$ is a polynomial.

The curve passes through the origin and touches the x-axis at the point $(3, 0)$

There is a maximum turning point at $(1, 2)$ and a minimum turning point at $(3, 0)$

On separate diagrams, sketch the curve with equation

(i) $y = 3f(2x)$

(3)

(ii) $y = f(-x) - 1$

(3)

On each sketch, show clearly the coordinates of

- the point where the curve crosses the y-axis
- any maximum or minimum turning points

2021 Jan P3

3.

$$f(x) = 3 - \frac{x-2}{x+1} + \frac{5x+26}{2x^2-3x-5} \qquad x > 4$$

(a) Show that

$$f(x) = \frac{ax+b}{cx+d} \qquad x > 4$$

where a, b, c and d are integers to be found.

(4)

(b) Hence find $f^{-1}(x)$

(2)

(c) Find the domain of f^{-1}

(2)

2021 June P3

4. The functions f and g are defined by

$$f(x) = \frac{4x + 6}{x - 5} \qquad x \in \mathbb{R},\ x \neq 5$$

$$g(x) = 5 - 2x^2 \qquad x \in \mathbb{R},\ x \leqslant 0$$

(a) Solve the equation

$$fg(x) = 3$$

(4)

(b) Find f^{-1}

(3)

(c) Sketch and label, on the same axes, the curve with equation $y = g(x)$ and the curve with equation $y = g^{-1}(x)$. Show on your sketch the coordinates of the points where each curve meets or cuts the coordinate axes.

(3)

2021 June P3

6. Given that k is a positive constant,

(a) on separate diagrams, sketch the graph with equation

(i) $y = k - 2|x|$

(ii) $y = \left|2x - \frac{k}{3}\right|$

Show on each sketch the coordinates, in terms of k, of each point where the graph meets or cuts the axes.

(4)

(b) Hence find, in terms of k, the values of x for which

$$\left|2x - \frac{k}{3}\right| = k - 2|x|$$

giving your answers in simplest form.

(4)

2021 Oct P3

1. The function f is defined by

$$\mathrm{f}(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \qquad x > 0$$

(a) Show that $\mathrm{f}(x) = \dfrac{5x}{x + 3}$

(3)

(b) Find f^{-1}

(3)

(c) (i) Find, in simplest form, $\mathrm{f}'(x)$.

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer.

(3)

2021 Oct P3

2.

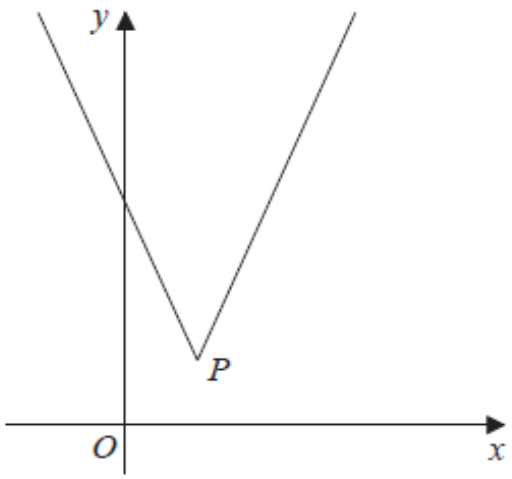

Figure 1

Figure 1 shows a sketch of part of the graph with equation $y = f(x)$, where

$$f(x) = |3x - 13| + 5 \qquad x \in \mathbb{R}$$

The vertex of the graph is at point P, as shown in Figure 1.

(a) State the coordinates of P.

(2)

(b) (i) State the range of f.

(ii) Find the value of $ff(4)$

(2)

(c) Solve, using algebra and showing your working,

$$16 - 2x > |3x - 13| + 5$$

(4)

The graph with equation $y = f(x)$ is transformed onto the graph with equation $y = af(x + b)$

The vertex of the graph with equation $y = af(x + b)$ is $(4, 20)$

Given that a and b are constants,

(d) find the value of a and the value of b.

(2)

2022 Jan P3

6. The function f is defined by

$$f(x) = \frac{5x - 3}{x - 4} \qquad x > 4$$

(a) Show, by using calculus, that f is a decreasing function.

(3)

(b) Find f^{-1}

(3)

(c) (i) Show that $ff(x) = \frac{ax + b}{x + c}$ where a, b and c are constants to be found.

(ii) Deduce the range of ff.

(5)

2022 Jan P3

7.

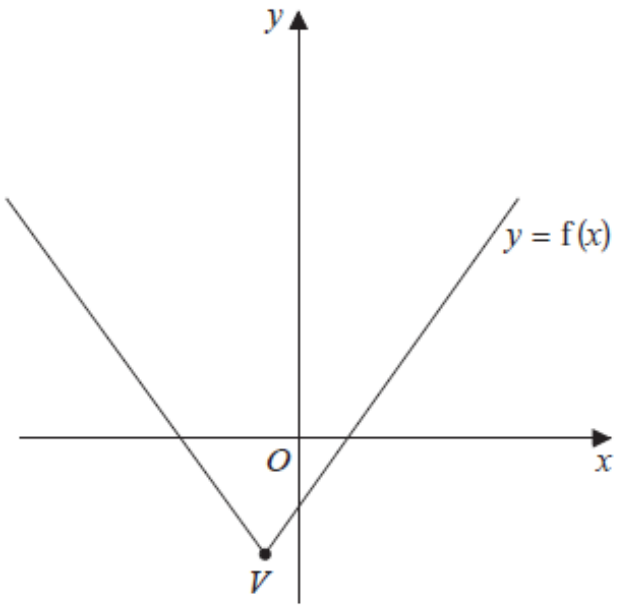

Figure 2

Figure 2 shows a sketch of part of the graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \frac{1}{2}|2x + 7| - 10$$

(a) State the coordinates of the vertex, V, of the graph.

(2)

(b) Solve, using algebra,

$$\frac{1}{2}|2x + 7| - 10 \geqslant \frac{1}{3}x + 1$$

(4)

(c) Sketch the graph with equation

$$y = |\mathrm{f}(x)|$$

stating the coordinates of the local maximum point and each local minimum point.

(4)

2022 June P3

2. The functions f and g are defined by

$$f(x) = \frac{5 - x}{3x + 2} \qquad x \in \mathbb{R}, x \neq -\frac{2}{3}$$

$$g(x) = 2x - 7 \qquad x \in \mathbb{R}$$

(a) Find the value of $fg(5)$

(2)

(b) Find f^{-1}

(3)

(c) Solve the equation

$$f\left(\frac{1}{a}\right) = g(a + 3)$$

(4)

2022 June P3

5.

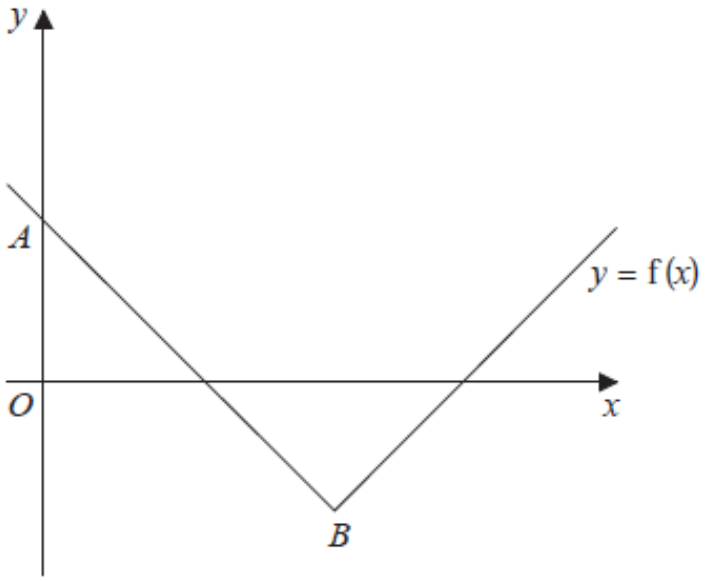

Figure 2

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \qquad x \in \mathbb{R}$$

and k is a positive constant.

The graph intersects the y-axis at the point A and has a minimum point at B as shown.

(a) (i) Find the y coordinate of A

(ii) Find, in terms of k, the x coordinate of B

(2)

(b) Find, in terms of k, the range of values of x that satisfy the inequality

$$|kx - 9| - 2 < 0$$

(3)

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of k

(3)

2022 Oct P3

2. The functions f and g are defined by

$$f(x) = 5 - \frac{4}{3x + 2} \qquad x \geqslant 0$$

$$g(x) = \left| 4\sin\left(\frac{x}{3} + \frac{\pi}{6}\right) \right| \qquad x \in \mathbb{R}$$

(a) Find the range of f

(2)

(b) (i) Find $f^{-1}(x)$

(ii) Write down the domain of f^{-1}

(3)

(c) Find $fg(-\pi)$

(2)

2022 Oct P3

7.

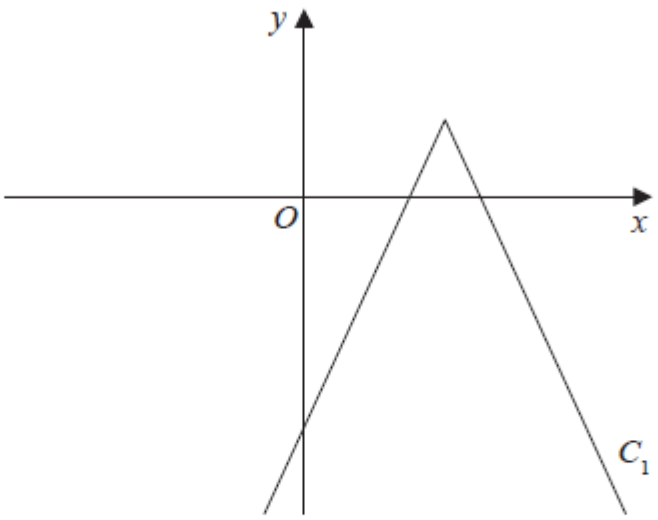

Figure 3

Figure 3 shows a sketch of the graph of C_1 with equation

$$y = 5 - |3x - 22|$$

(a) Write down the coordinates of

(i) the vertex of C_1

(ii) the intersection of C_1 with the y-axis.

(2)

(b) Find the x coordinates of the intersections of C_1 with the x-axis.

(2)

Diagram 1, shown on page 21, is a copy of Figure 3.

(c) On Diagram 1, sketch the curve C_2 with equation

$$y = \frac{1}{9}x^2 - 9$$

Identify clearly the coordinates of any points of intersection of C_2 with the coordinate axes.

(3)

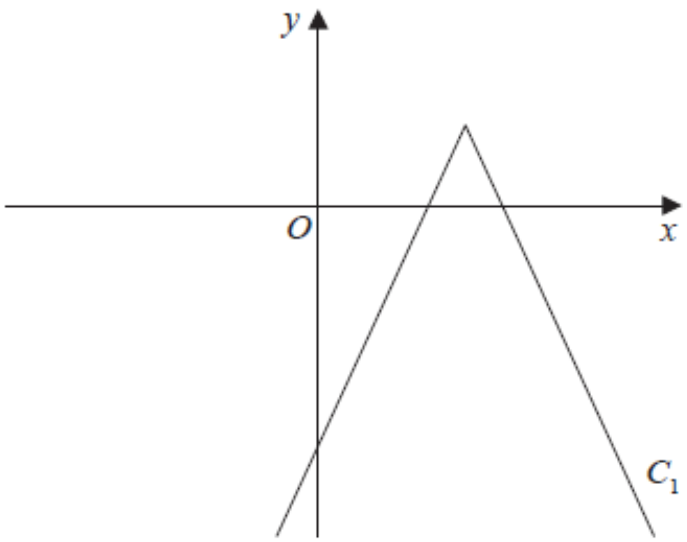

Diagram 1

2023 Jan P3

1. The functions f and g are defined by

$$f(x) = 9 - x^2 \qquad x \in \mathbb{R} \quad x \geqslant 0$$

$$g(x) = \frac{3}{2x+1} \qquad x \in \mathbb{R} \quad x \geqslant 0$$

(a) Write down the range of f

(1)

(b) Find the value of $fg(1.5)$

(2)

(c) Find g^{-1}

(3)

2023 Jan P3

6.

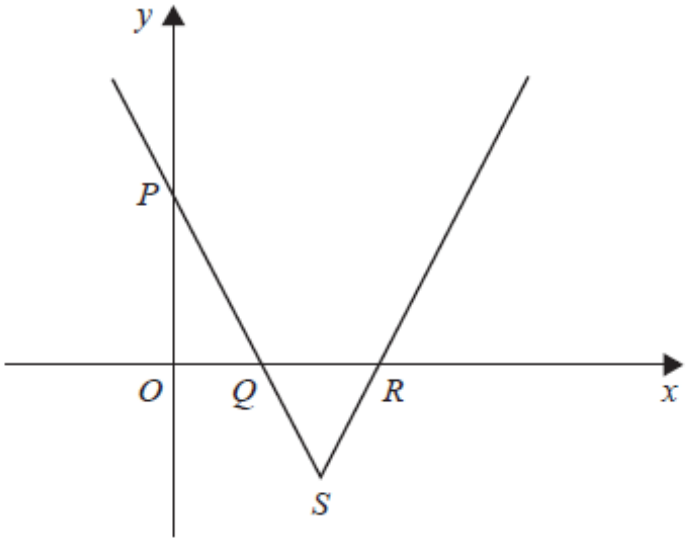

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = |3x - 5a| - 2a$$

where a is a positive constant.

The graph

- cuts the y-axis at the point P
- cuts the x-axis at the points Q and R
- has a minimum point at S

(a) Find, in simplest form in terms of a, the coordinates of

(i) point P

(ii) points Q and R

(iii) point S

(4)

(b) Find, in simplest form in terms of a, the values of x for which

$$|3x - 5a| - 2a = |x - 2a|$$

(4)

2023 June P3

4. The function f is defined by

$$f(x) = 2x^2 - 5 \qquad x \geqslant 0 \quad x \in \mathbb{R}$$

(a) State the range of f

(1)

On the following page there is a diagram, labelled Diagram 1, which shows a sketch of the curve with equation $y = f(x)$.

(b) On Diagram 1, sketch the curve with equation $y = f^{-1}(x)$.

(2)

The curve with equation $y = f(x)$ meets the curve with equation $y = f^{-1}(x)$ at the point P

Using algebra and showing your working,

(c) find the exact x coordinate of P

(3)

Question 4 continued

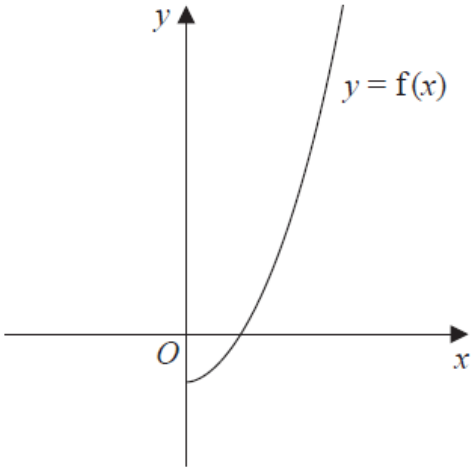

Diagram 1

2023 June P3

6.

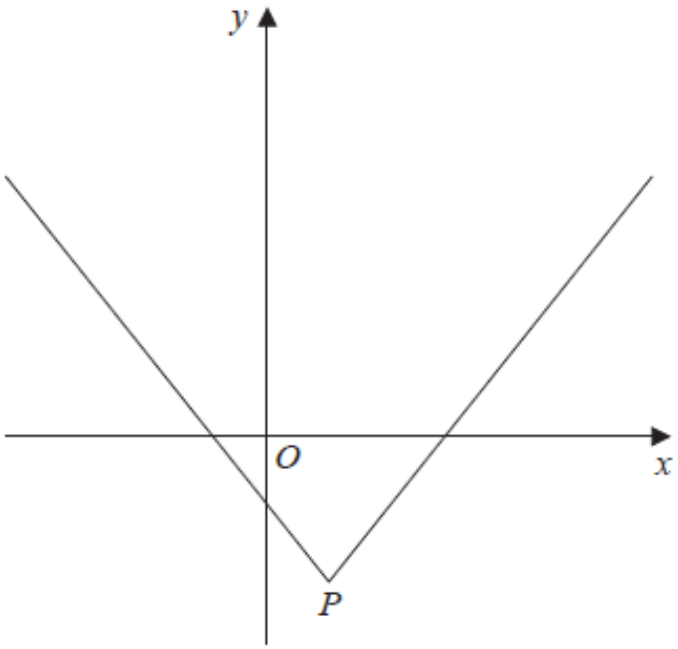

Figure 2

Figure 2 shows a sketch of the graph $y = f(x)$, where

$$f(x) = 3|x - 2| - 10$$

The vertex of the graph is at point P, shown in Figure 2.

(a) Find the coordinates of P

(2)

(b) Find $ff(0)$

(2)

(c) Solve the inequality

$$3|x - 2| - 10 < 5x + 10$$

(2)

(d) Solve the equation

$$f(|x|) = 0$$

(3)

2023 Oct P3

2. The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find $ff(6)$

(2)

(b) Find f^{-1}

(3)

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of a for which

$$gf(a) = 7$$

(3)

2023 Oct P3

9. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

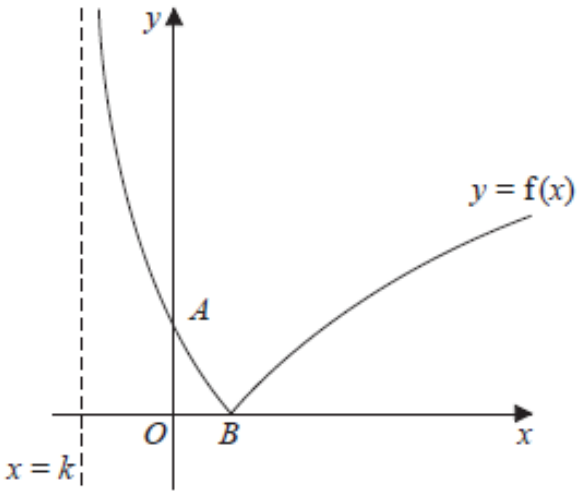

Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \qquad x > k$$

where k is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the y-axis at point A
- meets the x-axis at point B

as shown in Figure 2,

(a) state the value of k

(1)

(b) (i) find the y coordinate of A

(ii) find the exact x coordinate of B

(3)

(c) Using algebra and showing your working, find the set of values of x such that

$$|2 - 4\ln(x + 1)| > 3$$

(5)

2024 Jan P3

1. The point $P(-4, -3)$ lies on the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$

Find the point to which P is mapped when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation

(a) $y = \mathrm{f}(2x)$

(1)

(b) $y = 3\mathrm{f}(x - 1)$

(2)

(c) $y = |\mathrm{f}(x)|$

(1)

2024 Jan P3

4. The function f is defined by

$$f(x) = \frac{2x^2 - 32}{3x^2 + 7x - 20} + \frac{8}{3x - 5} \qquad x \in \mathbb{R} \quad x > 2$$

(a) Show that $f(x) = \dfrac{2x}{3x-5}$

(3)

(b) Show, using calculus, that f is a decreasing function.
You must make your reasoning clear.

(3)

The function g is defined by

$$g(x) = 3 + 2\ln x \qquad x \geqslant 1$$

(c) Find g^{-1}

(3)

(d) Find the exact value of a for which

$$gf(a) = 5$$

(4)

2024 Jan P3

8.

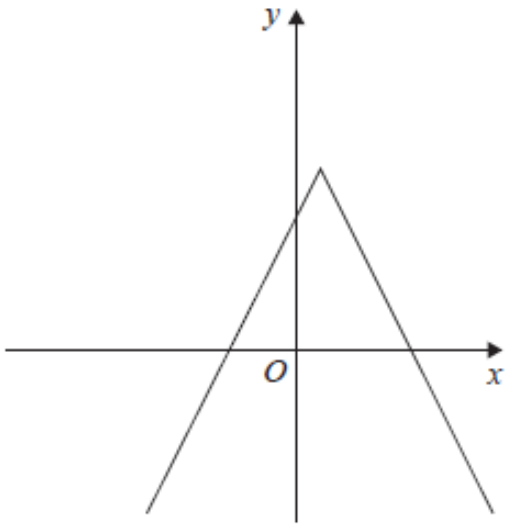

Figure 2

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

The graph shown in Figure 2 has equation

$$y = a - |2x - b|$$

where a and b are positive constants, $a > b$

(a) Find, giving your answer in terms of a and b,

(i) the coordinates of the maximum point of the graph,

(ii) the coordinates of the point of intersection of the graph with the y-axis,

(iii) the coordinates of the points of intersection of the graph with the x-axis.

(5)

On page 24 there is a copy of Figure 2 called Diagram 1.

(b) On Diagram 1, sketch the graph with equation

$$y = |x| - 1$$

(2)

Given that the graphs $y = |x| - 1$ and $y = a - |2x - b|$ intersect at $x = -3$ and $x = 5$

(c) find the value of a and the value of b

(4)